Research Statement

As a physicist working in a mathematics institute, I have succesfully finished projects with both physicists and mathematicians. In the last couple of years, I have been able to use the breadth of techniques I acquired in my Ph.D., where I studied the building blocks of string scattering amplitudes. Auspiciously, the mathematical tools that work on string scattering amplitudes also work on Feynman integrals and cosmological correlators – they are all twisted periods. I have worked on making this fortunate fact effective.

In ref. [1] I checked the so-called coaction and double-copy properties for a family of genus-zero integrals that universally appear in tree-level string amplitudes and Feynman integrals. This motivated my work in refs. [2, 3], where I derived formulas for the coaction and single-valued maps of multiple polylogarithms. In refs. [4, 5], I studied integrals over punctured Riemann surfaces of genus $g \geq 1$, and found that, like for tree-level string amplitudes, they satisfy double-copy relations. My work [6] introduces border bases for the rational Weyl algebra and, as an immediate application, these allow us to recover ideals of annihilating operators for Feynman integrals (or cosmological correlators, string integrals, etc...) from their differential equations.

Genus zero coaction and double-copy

In my work [1] I check the coaction conjecture of ref. [7] that initially arose from a particle-physics context, and which can also be framed in terms of string integrals. The integrals studied in this work are functions of punctures z_i, and their α'-expansion is given in both multiple zeta values (MZVs) and multiple polylogarithms (MPLs). The coaction acts nontrivially on both of these. In order to obtain the α'-expansion in the first place, I used a differential equation on a vector of "master integrals" (i.e. a vector of integrals that closes under taking a differential equation) for this setup. I also studied for a closed-string version of these integrals how the KLT-formula form of the double copy relates to the single-valued map. The language of twisted (co)homology [8, 9] proved useful in this genus-zero setup.

The interplay between the coactions of MZVs and MPLs I saw in [1] motivated my work [2, 3]. Here, I studied generating functions of MZVs and MPLs in several variables z_i. The punchline is that one can conveniently formulate a formula for the coaction (and the single-valued map) of MPLs purely in their generating functions. Recapping, a conjecture in particle physics motivated my work in string integrals, which further motivated me to conjecture (and prove!) mathematical statements for MPLs.

Integrals over punctured Riemann surfaces

In my work [4, 5] I studied integrals closely related to g-loop string amplitudes in the chiral splitting formalism, where one leaves the loop momenta ℓ_J, surface moduli Ω_{IJ} and all but one puncture unintegrated. The main result of this study was that these integrals satisfy a version of the KLT relations: the double-copy formula that relates closed-string amplitudes to a bilinear combination of open-string amplitudes. Moreover, I developed a novel technique to numerically evaluate the integration kernels of higher-genus polylogarithms – also known as Enriquez kernels [10, 11, 12]. This expanded on the works of mathematicians [13, 14, 15] to obtain results of physical interest.

Border bases in the rational Weyl algebra

In my work [6], I define and characterize border bases – a generalization of Gröbner bases – for the rational Weyl algebra R_n, a noncommutative ring of differential operators. These border bases describe ideals $J \subset R_n$ of finite holonomic rank – examples of such ideals include the ideals of annihilating differential operators for Feynman integrals, cosmological correlators, and the string integrals of [1]. As an application, starting from the differential equation of a Feynman integral F we are able to recover an ideal of differential operators J that annihilate F. This bridges a gap between R_n-module and Feynman integral communities. More concretely, this allows us to study annihilating ideals of Feynman integrals with the highly optimized algorithms that compute their differential equations.

Further directions

The explorations in my previous work opened up conversations with both mathematicians and physicists to tackle problems we're both interested in. One project that is ongoing is the following:

1. Given a Schottky group G that uniformizes a compact Riemann surface of genus-g, how can one recover the corresponding algebraic curve? This can be done numerically via the so-called canonical map, which maps the Riemann surface into an algebraic curve in P^{g-1}. The same numerical techniques I employed in my work [5], allows one to effectively perform this up to genus 6. This is ongoing work with Samantha Fairchild and Ángel David Ríos Ortiz.

There are several projects at the interface of QFT, string theory, number theory, algebraic geometry and machine learning that I believe are feasible in the near future. Some concrete starting points for my future research include the following.

1. Feynman integrals and cosmological correlators are all twisted periods, i.e. pairings of (relative) twisted homology and cohomology groups [16]. In order to study the coaction conjecture of [7], one needs to understand both of these groups for Feynman integrals. I plan to study more extensively the twisted *homology* groups (i.e. the integration contours) of Feynman integrals. As a first step in this project, I will study these homology groups in different integral representations, e.g. Baikov and Feynman parametrization.

2. From the string-theory side, how does one go from the results of my works [4, 5] to integrals over all the puncture positions at genus $g \geq 1$? An answer for $g = 1$ is given by [17]. This suggest that a careful reading of ref. [17] can lead the path to an all-loop KLT formula relating closed-string to open-string amplitudes.

3. In my work [5] I made use of a novel method to evaluate the Enriquez kernels – the integrands of polylogarithms on a genus-g Riemann surface – of [10, 11, 12]. This numerical toolkit can be systematized and expanded. Notably, this method (and the one of [10]) requires the Schottky uniformization of a Riemann surface. Thus, a first step is to develop a method to find (numerical) Schottky uniformizations of non-hyperelliptic Riemann surfaces, which would interest both mathematicians and physicists.

4. Schottky groups uniformize non-degenerate algebraic curves. In the boundary of Schottky groups, one can reach singular curves – for example, union of lines. In my numerical experiments, I have found certain families of Schottky groups that exhibit this kind of degeneration. It would be interesting to study these families in a systematic way, using tools of tropical geometry. A first step in this project requires implementing the algorithms of [18] with Schottky groups defined over Puiseux series, $\mathrm{SL}\,(2, \mathbb{C}((\epsilon)))$.

5. It's a known fact that compact Riemann surfaces (and algebraic curves) admit an uniformization by a Schottky group. A numerical approach for this exists in the literature for real hyperelliptic curves [19]. Numerical Schottky uniformization can be addressed via machine learning methods, as data can be obtained, for example, for the non-hyperelliptic families curves studied in [20]. More interestingly, via methods of geometric deep learning, one can study the analytic manifold of Jacobians of curves, embedded in the space of principally polarized Abelian varieties (i.e. the solution to the Schottky problem: which period matrices corresponds to curves).

6. The border bases of my work [6] allow us to explore the ideals of annihilating differential operators of Feynman integrals and cosmological correlators. This is particularly interesting for cosmological correlators, where there are combinatorial formulas for their differential equations, coming form the so-called kinematic flow [21]. It would be interesting to see if either (a) the symmetries of cosmological correlators or (b) the combinatorial aspects of the kinematic flow are reflected in their annihilating ideals. Furthermore, it is known that cosmological correlators are restrictions of GKZ systems (see e.g. [22]). With these annihilating ideals in hand, we can readily determine whether or not these ideals have similar properties to the ideals of GKZ systems.

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