Research Statement

During my PhD I have studied the building blocks of string scattering amplitudes, both for their expansion around their field-theory limit and for their number-theoretic properties. These are special functions that appear in computing certain integrals over the configuration spaces of Riemann surfaces. In ref. [1] I checked the so-called coaction conjecture for a family of genus-zero integrals that universally appear in tree-level string amplitudes and Feynman integrals, and other properties of these such as their double copy and analytic continuation. My work [2] extended some results of [1] to a genus-one setup, i.e. prototype integrals in one-loop string amplitudes: I found a differential equation method that allows the evaluation of their low-energy expansion, and their analytic continuation. In ref. [3] I expanded the toolkit available for genus-one amplitudes to also work for amplitudes of e.g. heterotic string theory.

Coaction and double-copy properties of configuration-space integrals at genus zero

In my work [1] I check the coaction conjecture of ref. [4] that initially arose from a particle-physics context and that can also be framed in terms of a "lazy" N-point string amplitude: instead of integrating over the $(N-3)$ puncture positions as prescribed, we only integrate over p of them. These integrals therefore are functions of the leftover $(N-p-3)$ cross ratios, their α'-expansion is given in both MZVs and MPLs, and the coaction acts non-trivially on both of these. In order to obtain the α'-expansion in the first place, I used a differential equation on a vector of "master integrals" (i.e. a vector of integrals that closes under taking a differential equation) for this setup. I also studied for a closed-string version of these integrals how the KLT-formula form of the double copy relates to the single-valued map. The language of twisted (co)homology [5, 6] proved useful in this genus-zero setup.

Open-string integrals with multiple unintegrated punctures at genus one

In my work [2], I obtained an α'-expansion of genus-one integrals and their generalizations to arbitrary numbers of unintegrated punctures from a differential equation. In a way, this work can be seen as a genus-one extension of some of the results in [1], or as an extension of the results in [7] to additional unintegrated punctures. I found that the proposed "master integrals" of [1] have a genus-one extension that also closes under taking a differential equation. This is interesting because there is (yet) no twisted cohomology argument for this (except for the case of [8]). Moreover, I found connections the so-called universal KZB equation [9]. In particular I found graded representations of the genus-one Kohno-Drinfeld algebra $\mathfrak{t}_{1,n}$ and novel analytic continuation identities for eMPLs coming from this differential equation.

Basis decompositions of genus-one string integrals

In my work [3], I obtained identities useful for the α'-expansion of, say, heterotic string amplitudes. More specifically, heterotic string amplitudes involve terms that are not obviously related to the vectors of "master integrals" of e.g [10]. Thus, one needs to do some work, via integration-by-parts (IBP) and the Fay identities (a genus-one version of partial-fractioning) to turn string integrands into our "master integrals". A punchline of this work is that the "master integrals" of previous work are sufficient, and thus it gives further justification in using the term "integral basis" when referring to them. The expansion coefficients are worked out in a variety of all-multiplicity cases, and it remains to clarify their relation to intersection numbers in a twisted cohomology framework.

Further directions

The explorations in my previous work opened up conversations with both mathematicians and physicists to tackle problems we're both interested in. The following are two ongoing projects that address some questions raised by my previous work.

1. The main big question is: what is the twisted cohomology framework for genus-one? Mathematicians [8, 11, 12] have been working on a twisted cohomology setup for genus-one integrals in which only one puncture is integrated. Further understanding why they make the choices they made and how they relate to string amplitudes (and crucially, what are the implications for BCJ double-copy as we take the field-theory limit?) is subject of ongoing work in collaboration with Rishabh Bhardwaj, Andrzej Pokraka, and Lecheng Ren.

2. We learned a lot from the setup of our work in [1], but... can we turn this idea around? If we assume that a generating function of MPLs satisfy the coaction principle - as a generating series - does this in turn give a recursion for the coaction of MPLs? Abstract lie algebra generators related to MZVs and the Drinfeld-Kohno algebra play a crucial role here, and both have concrete realizations in the initial values and differential equations, respectively, of the configuration space integrals in [1]. This is ongoing work with Hadleigh Frost, Martijn Hidding, Deepak Kamlesh, Oliver Schlotterer and Bram Verbeek.

There are several other projects at the interface of QFT, string theory, number theory and algebraic geometry I would enjoy working on in the near future. More concrete starting points for my future research include the following.

1. What does a one-loop version of the double-copy look like for string-theory amplitudes? One candidate, by twisted cohomology, would be an extension of [8, 11, 12] to multiple integrated punctures. Another approach is the recent work of Stieberger on one-loop KLT relations [13]. Is there a framework where both of these approaches fit in?

2. There are versions of coaction formulas at genus-one and higher, in refs. [14, 15, 16]. If the genus-one configuration-space integrals of my work [2] are given a coaction via twisted cohomology[17], this should imply a concrete realization of de Rham periods for the genus-one coactions on eMPLs, by α'-expanding the integrals and comparing coefficients.

3. I was able to write interesting properties of the analytic continuation of eMPLs in my work [2] by using the properties of the universal KZB equation. However, there are many more properties, such as A-cycle and B-cycle translation of punctures, and S- and T-transformations of the surface moduli τ that are implicit in the work of [9]. I plan to spell this out for both efficient numerical implementation and analytic understanding of eMPLs.

4. A proposal for the integration kernels in higher-genus polylogs was described in [18] in a language very amenable for computations. It would be interesting to use these building blocks to propose configuration-space integrals for higher-genus string amplitudes. I plan to study such integrals as an extension to the twisted cohomology setup of [8], and compare this to the higher-genus approaches of [19, 20].

5. I plan to extend the numerical toolkit to evaluate the functions in [18] defined on genus $g > 1$ Riemann surfaces. This would help for both exploring and checking relations (e.g. double-copy relations) for higher-genus configuration-space integrals. The Schottky parametrization seems key here [21]. As a first step, I plan to port [21] from MATLAB to Julia.

References

[1] R. Britto, S. Mizera, C. Rodriguez and O. Schlotterer, *Coaction and double-copy properties of configuration-space integrals at genus zero*, *JHEP* **05** (2021) 053 [2102.06206].

[2] A. Kaderli and C. Rodriguez, *Open-string integrals with multiple unintegrated punctures at genus one*, *JHEP* **10** (2022) 159 [2203.09649].

[3] C. Rodriguez, O. Schlotterer and Y. Zhang, *Basis decompositions of genus-one string integrals*, 2309.15836.

[4] S. Abreu, R. Britto, C. Duhr and E. Gardi, *Algebraic Structure of Cut Feynman Integrals and the Diagrammatic Coaction*, *Phys. Rev. Lett.* **119** (2017) 051601 [1703.05064].

[5] F. Brown and C. Dupont, *Single-valued integration and double copy*, *J. Reine Angew. Math.* **2021** (2021) 145 [1810.07682].

[6] S. Mizera, *Aspects of Scattering Amplitudes and Moduli Space Localization*, Ph.D. thesis, Princeton, Inst. Advanced Study, 2020. 1906.02099. 10.1007/978-3-030-53010-5.

[7] J. Broedel, A. Kaderli and O. Schlotterer, *Two dialects for KZB equations: generating one-loop open-string integrals*, *JHEP* **12** (2020) 036 [2007.03712].

[8] T. Mano and H. Watanabe, *Twisted cohomology and homology groups associated to the riemann-wirtinger integral*, *Proceedings of the American Mathematical Society* **140** (2012) 3867.

[9] D. Calaque, B. Enriquez and P. Etingof, *Universal kzb equations: The elliptic case*, in *Algebra, Arithmetic, and Geometry: Volume I: In Honor of Yu. I. Manin*, Y. Tschinkel and Y. Zarhin, eds., (Boston), pp. 165–266, Birkhäuser Boston (2009), DOI.

[10] C.R. Mafra and O. Schlotterer, *All Order α' Expansion of One-Loop Open-String Integrals*, *Phys. Rev. Lett.* **124** (2020) 101603 [1908.09848].

[11] S. Ghazouani and L. Pirio, *Moduli spaces of flat tori and elliptic hypergeometric functions*, 1906.11857.

[12] Y. Goto, *Intersection numbers of twisted homology and cohomology groups associated to the riemann-wirtinger integral*, 2206.03177.

[13] S. Stieberger, *A Relation between One-Loop Amplitudes of Closed and Open Strings (One-Loop KLT Relation)*, 2212.06816.

[14] J. Broedel, C. Duhr, F. Dulat, B. Penante and L. Tancredi, *Elliptic symbol calculus: from elliptic polylogarithms to iterated integrals of Eisenstein series*, *JHEP* **08** (2018) 014 [1803.10256].

[15] M. Wilhelm and C. Zhang, *Symbology for elliptic multiple polylogarithms and the symbol prime*, *JHEP* **01** (2023) 089 [2206.08378].

[16] A. Forum and M. von Hippel, *A Symbol and Coaction for Higher-Loop Sunrise Integrals*, *SciPost Phys.* **6** (2023) 050 [2209.03922].

[17] S. Abreu, R. Britto, C. Duhr, E. Gardi and J. Matthew, *From positive geometries to a coaction on hypergeometric functions*, *JHEP* **02** (2020) 122 [1910.08358].

[18] E. D'Hoker, M. Hidding and O. Schlotterer, *Constructing polylogarithms on higher-genus Riemann surfaces*, 2306.08644.

[19] H. Watanabe, *Twisted cohomology of a punctured riemann surface*, *Kumamoto J. Math.* **29** (2016) 55.

[20] H. Watanabe, *Twisted cohomology of the complement of theta divisors in an abelian surface*, *International Journal of Mathematics* **27** (2016) 1650049.

[21] D.G. Crowdy and J.S. Marshall, *Computing the schottky-klein prime function on the schottky double of planar domains*, *Computational Methods and Function Theory* **7** (2007) 293.